

# Using Models to Support Understanding in Mathematics Pre-Service Teachers

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Reflecting on the mathematics classes I experienced as a K-12 student, I remember sitting quietly in a row, facing the front of the room as my teacher scrawled seemingly endless examples on the board. I remember participating in an unrelenting cycle of notes-homework-quiz-test. Even so, I was unbothered by the monotony. My grades were high, and I entered college feeling like a competent mathematician. My “competence,” unbeknownst to me, was shallow and inflexible. My undergraduate coursework opened my eyes to a new way of learning and teaching mathematics, one that involved complex problem solving, visual representation, and intentional collaboration. Through the use mathematical models, manipulatives, and purposeful discourse, mathematics came alive as I began to deeply explore, understand, and think critically about every concept I took for granted. I graduated and began teaching seventh grade thinking that I had finally figured it all out, that I had at last solved the mystery of mathematics.

Of course, I was egregiously wrong. During my years teaching middle school math, I was always delighted to understand the various ways my students solved problems. Their strategies challenged me to deepen my own conceptual understanding. As a mathematics education doctoral student, my conceptual understanding continues to grow more robust through readings, collaborations with peers, and my own work with pre-service teachers (PSTs). Indeed, there is always more to discover and more connections to be made within the world of mathematics. In our work as mathematics teacher educators (MTEs), we must be continually willing to engage our minds as mathematicians and do the same with pre-service teachers. In doing so, we can help support their development as creative, thoughtful, and adaptive thinkers. These are the kinds of teachers we need in an ever-changing world.

## Review of Literature

Even though standards have increasingly advocated for the use of mathematical modeling and visual representation, many PSTs do not enter preparation programs with a robust conceptual understanding or experience these opportunities during their college coursework. In fact, in a study of teacher preparation and mathematical content courses, less than half report the regular use of visual representation or modeling (Jung et al., 2019). Education students, while typically able to solve mathematics problems using formulas or procedures, often do not understand why those methods are successful (Obara, 2009). As a result, many PSTs, especially those slated to teach elementary, experience low self-efficacy in their ability to teach mathematical content to students (Hine, 2015).

The use of visual models and various representations is critical for the development of deep, conceptual understanding. In interviews, PSTs identified that the most formative experiences of their preparation centered around the use of manipulatives and visual models (Jung et al., 2019). Other studies indicate that the use of visual models, including the use of technology is fundamental in the development of mathematical creativity and flexible understanding of content (Bicer, 2021; Hine, 2015; Obara, 2009). According to the National Council of

Teachers of Mathematics (NCTM) (2014), effective teachers use visual representations to help students solidify their mathematical understanding. Therefore, MTEs must use models and visual representation to engage PSTs as learners of mathematics, working to gain insight into their own mathematical understanding that will enhance their future instruction (Association of Mathematics Teacher Educators, 2017).

### Dividing Fractions with PSTs

During a methods course, I sought to challenge the “keep, change, flip” method of dividing fractions

1.  $6 \div 2$

2.  $5 \div \frac{1}{2}$

3.  $5 \div \frac{2}{3}$

4.  $1\frac{1}{2} \div \frac{2}{3}$

with my PSTs. I engaged them in an exploration of scaffolded questions and laid several manipulatives on their tables including fraction tiles, circles, Cuisenaire rods, graph paper, rulers, color tiles, pattern blocks, etc. The questions included:

With the exception of the first problem, they struggled to be able to conceptually divide without the use of procedures, tricks, or converting the numbers into decimals. Utilizing NCTM’s Math Teaching Practices (2014) of purposeful questioning and the support of productive struggle, I pushed them to engage as mathematics learners. What did they know? How could they utilize tools and visuals to support their thinking? How could collaboration help support or even deepen their understanding?

I am happy to report that they did experience success. They began to talk to each other, draw pictures, use the tools, make mistakes, and try again. They were creatively problem solving – the exact practice we want to see in PK-12 students. At the end of the class session, one of them stated, “My brain hurts. I’ve never thought so deeply about fractions before, but I feel like I actually understand them now.” Another said, “I can’t wait to tell my roommate!” This simple experience increased their self-efficacy and deepened their concept of fractions.

### Significance and Conclusion

Effective teachers use visual representations to help students solidify their mathematical understanding and increase their ability to think flexibly. Reflecting on how I learned mathematics, many of these visual models were absent. Perhaps they are also absent for some of the pre-service teachers entering educator preparation programs. Therefore, MTEs must engage their PSTs in experiences that support understanding through the use of visual representation. How can pre-service teachers be expected to design visually rich learning opportunities for students if they themselves have never experienced such classrooms? Mathematics teacher preparation programs must use models and visual representation to engage pre-service teachers as learners of mathematics, working to gain insight into their own mathematical understanding that will enhance their future instruction (Association of Mathematics Teacher Educators, 2017). This is not only critical for their own mathematical development, but also has implications for their future classrooms. If mathematicians at varying levels benefit from the use of models, teacher educators cannot assume that pre-service teachers are above the use of drawings, manipulatives, graphs, or charts to help them make meaning of mathematical concepts. It is crucial for teacher educators to provide pre-service teachers the same kinds of experiences we want them to use in their future classrooms.

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