Preservice Teachers' Use of Representations in a Modeling Task on Area and Perimeter

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Representations are mediums that convey understanding, support critical thinking, and aid in problem solving. These mediums apply to students' understanding and their application of mathematical modeling (MM) that is a process of using representations and mathematics as tools to understand real-word problems, discover relationships among variables, justify solution strategies as well as share and critique these strategies (Asempapa et al., 2017; Lesh & Doerr, 2003; Suh et al., 2021).

In several mathematics teacher preparation programs in the United States, there is a limited focus on MM (Asempapa et al., 2017; Paulocci & Wessels, 2017). This suggests that many preservice teachers (PSTs) have not had opportunities to engage in MM and may not adequately understand the actions associated with MM and the components within (Phillips, 2016). This limited understanding of MM becomes problematic when PSTs encounter students using representations to test, revise, and express their mathematical thinking (Lesh & Zawojewski, 2007; Moore et al., 2013).

Real-life concepts such as perimeter and area are important for developing and improving students' understanding of mathematics (Moyer, 2001). However, research indicates most PSTs have a poor conceptual understanding of area and perimeter as well as a reliance on formulas (Moyer, 2001). As emphasized in preparation standards (e.g., AMTE, 2020) and content standards (e.g., National Governor's Association and the Council of Chief State School Officers [NGA-CCSSO], 2010), PSTs must be familiar with how different representations are useful in supporting students' understanding of mathematics. Therefore, this article provides a short description and results of an investigation into elementary and middle school PSTs' use and connection of mathematical representations when solving an area and perimeter MM task.

Methods

Thirty-one elementary and middle school PSTs were recruited from two, four-year university mathematics methods courses. Data consisted of participants' responses to a four-part modeling task on designing practical tables, which explores area and perimeter concepts, use of representations, and application of MM (Figure 1). Data analysis began with creating and refining a rubric, and then applying this rubric to the 29 of 31 participants (2 nonrespondents). Special focus was paid to the types and number of representations used by PSTs in the modeling process. Rating consistency was measured by the intraclass correlation coefficient which were all above 0.90 (Liljequist et al., 2019).

Figure 1
MM Task: A Table for 14!

A dinner is being held to honor Sir Jones of Little Village. You are given the task to design a rectangular table for 14 guests, including Sir Jones. The table you design has to be practical and accommodate all guests.

A. Explain how you will design this table using any representation. Take note of any simplifications or assumptions made.
B. The chef has prepared too much food. Now, the table must be redesigned to maximize the area. Justify why the table that you designed has maximized the area.
C. The food delivery truck is broken and cannot be fixed. Now, only snacks can be served. The table must be redesigned to minimize the area. Justify why the table that you designed has minimized the area.
D. Suppose you purchase a ribbon to decorate the edges of the tables designed in parts A-C above, what would you notice? How much ribbon would you need?

(Adapted from Asempapa et al., 2017)

Results

We summarize our findings by presenting and then discussing (a) the types and number of representations and (b) the characteristics that embody these representations. As presented in Table 1, PSTs used pictorial representations the most in Parts A and C (93.1% and 69.2%), and verbal representations were mostly used in Parts B and D (73.1% and 86.4%). None of the PSTs used symbolic representations alone except in Part D. Instead, they used symbolic representations paired with other representation(s). Only one PST used a pictorial representation that was paired with a verbal representation for Part D. Furthermore, Table 1 shows that many PSTs used only one or two representations. Approximately half of the PSTs used two representations in their solution process to solve Parts A and B (55.2% and 46.2%) and only one representation in their solution process for Parts C and D (50% and 54.5%). None of the PSTs used three representations for Part D.

Table 1

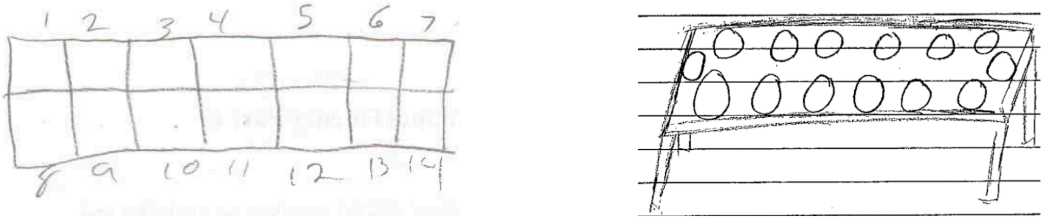
Type and Number of Representations Used in Each Part of the Task

	Part A (n = 29)	Part B (n = 26)	Part C (n = 26)	Part D (n = 22)
S	0 (0%)	0 (0%)	0 (0%)	3 (13.6%)
P	9 (31%)	2 (7.7%)	5 (19.2%)	0 (0%)
V	1 (3.4%)	7 (26.9%)	8 (30.8%)	9 (40.9%)
S & P	10 (34.5%)	5 (19.2%)	6 (23.1%)	0 (0%)
S & V	1 (3.4%)	1 (3.8%)	0 (0%)	9 (40.9%)
P & V	5 (17.2%)	6 (23.1%)	2 (7.7%)	1 (4.5%)
S, P, & V	3 (10.3%)	5 (19.2%)	5 (19.2%)	0 (0%)

Note. S = Symbolic Representation; P = Pictorial Representation; V = Verbal Representation. Non-respondents were excluded, and no PST used four representations.

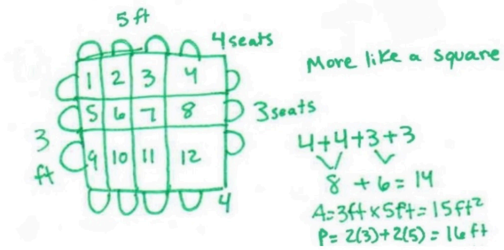
Upon examining the representations, we also noticed a lack of details and justifications. With respect to Part A, Figure 2 serves as an example of how participants used a limited number of representations by drawing a rectangular-shaped table and omitting dimensions or units of measure. On the other hand, as portrayed in Table 1 and shown in Figure 3, few PSTs used three types of representations. Additionally, very few recorded their assumptions when completing Parts A–C. One participant, however, did provide an assumption to the table's construction in Part A and calculation of perimeter in Part D: "I would give a variable for each guest, G. Then I would multiply that by 14. The total perimeter of my table would have to equal 14G" (Participant 19). Another PST discussed the arrangement of guests, which was evident in the table design: "I drew a table that was long enough for six people to fit on each side with two people on the end [and,] this table was very practical and can easily be built" (Figure 2, Participant 26). In general, participants did not propose whether their solution process was "good enough" and reasonable; they only addressed that the number of guests or the conditions in Parts B and C. Overall, the data showed that our PSTs struggled using multiple forms of representations in their MM engagement, including justifying how their table designs minimized or maximized the area.

Figure 2
Vague Pictorial Representation Examples for Part A



Note. Left panel: Participant 1's drawing. Right panel: Participant 26's drawing.

Figure 3
Three Types of Representations Example for Part B



Conclusion

Students are able to solve problems and communicate their mathematical ideas when they use representations and flexibly move between different representations (Bostic & Pape, 2010; Suh & Moyer, 2007). The ability to model problems requires the diverse use of representations. In this study, PSTs' limitation to the practice "use and connect mathematical representations" may result from an underdeveloped skill of seeing and representing mathematics in different forms. Most PSTs conceptualized area as a covering by drawing a single rectangle or a collection of smaller rectangles. Research shows that MM requires formulating questions and making decisions and assumptions (Suh et al., 2021; Zbiek, 2016). These actions in turn can impact the type of representations used. As described, the PSTs' solution process did not address the MM process (e.g., a lack of detail and justification), which signals a weak understanding of MM. It is critical that teacher educators foster practices that assist PSTs to develop an in-depth understanding of area and perimeter concepts that incorporates multiple representations. A high-leverage practice should provide PSTs purposeful and regular opportunities to see and represent mathematics in different forms and move between representations, which are part of the MM process.

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